

Quasinormal modes of semiclassical electrically charged black holes.

Owen Pavel Fernandez Piedra^{1a}

¹ *Departamento de Física y Química, Facultad de Mecánica,
Universidad de Cienfuegos, Carretera a Rodas,
km 4, Cuatro Caminos, Cienfuegos, Cuba*

Jeferson de Oliveira^{2b}

² *Instituto de Física, Universidade de São Paulo,
CP 66318, 05315-970, São Paulo, Brazil*

Abstract

We report the results concerning the influence of vacuum polarization due to quantum massive vector, scalar and spinor fields on the scalar sector of quasinormal modes in spherically symmetric charged black holes. The vacuum polarization from quantized fields produces a shift in the values of the quasinormal frequencies, and correspondingly the semiclassical system becomes a better oscillator with respect to the classical Reissner-Nordström black hole.

^a Email:opavel@ucf.edu.cu

^b Email:jeferson@fma.if.usp.br

I. INTRODUCTION

Quantum theory and General Relativity are two cornerstones of modern physics that for more than a century have contributed to increase our knowledge of the Universe as never before in the human history. With the help of the Quantum Theory we can explain micro-world phenomena, and the General Theory of Relativity allows us a deep understanding of the Universe at cosmological scales. Unfortunately, these two beautiful theories resist all attempts to bring them together. A unified theory of gravity and the quantum world would be very important to describe, for example, the origin of the Universe and its later development.

There are other simple phenomena that can be very interesting to describe by the future Quantum Gravity. Among other things, from the classical side, it is well known that the response of a black hole to small perturbations at intermediate times is characterized, under suitable boundary conditions, by a discrete set of complex frequencies called quasinormal frequencies, that depend only upon the parameters of the black hole [1–4]. From the quantum side, it would be interesting to see what changes appear in the evolution of quantum black holes under perturbations. Specially interesting is the behavior at intermediate times dominated by quasinormal response, because apart from allowing us to gain some valuable information about these objects, the quasinormal spectrum permits investigation of the black hole stability against small perturbations. Several numerical methods have been developed to study such an interesting problem [5–7].

Quasinormal modes appear to be important in other contexts, as for example, the AdS/CFT correspondence, where the inverse of imaginary part of quasinormal frequencies of AdS black holes can be interpreted as the dual CFT relaxation time [8] [9].

In a previous paper we considered the influence of vacuum polarization effects due to the backreaction of a quantum massive scalar field of large mass upon the quasinormal modes of electrically charged black hole solutions obtained solving the semiclassical Einstein field equations, with the quantum renormalized stress tensor of the quantized matter field as a source [10]. Such an influence appears essentially as an appreciable shift in the quasinormal frequencies that decreases as the bare black hole mass increases, and that do not have a strong dependence upon the quantum field parameters, leading to the conclusion that the quantum corrected black holes are less oscillatory with respect to its classical counterparts. Another

previous work along similar lines was done by Konoplya [11], for the BTZ black hole dressed by a massless scalar field, but in this case he considered the influence of particle creation around the event horizon, an effect that dominates over the vacuum polarization effect for massless fields.

To solve the backreaction problem in semiclassical gravity, we need to know the functional dependence of the renormalized stress energy tensor of the quantum field surrounding the classical compact object on a wide class of metrics [12]. Unfortunately, this is a very difficult problem, and up to now, there exist only approximate methods to develop a tractable expression for this quantity [12–19]. Since the pioneering work of York [20], who solved the semiclassical Einstein equations for a Schwarzschild black hole dressed by a massless conformally coupled scalar field, using for the quantum stress energy tensor the results given earlier by Page [13], there are some related works in the literature, both for massless and massive quantum fields of different values of the spin parameter. To see the effects of the backreaction upon the black hole response to small perturbations, quantum massless fields as sources of the quantum corrections are not the most suitable candidates, because the semiclassical metric components diverge as $r \rightarrow \infty$ and to obtain the correct solutions to the backreaction problem we need to impose some sort of boundary to the system under study, a feature that causes a change the quasinormal spectrum. A different situation happens in the case of very massive fields, for which the vacuum polarization effects are not difficult to compute constructing the quantum stress energy tensor by means of the Schwinger-DeWitt expansion of the quantum effective action, whenever the Compton's wavelength of the field is less than the characteristic radius of curvature [15, 16, 18, 21–23].

It is important to mention that in semiclassical gravity the unavoidable effects due to the metric fluctuations and the associated graviton contributions to the complete quantum stress-energy tensor are ignored, a fact that is usually justified considering that there exists a regime in which the gravitational field can be regarded as a classical entity, and the effects of the remaining matter fields after quantization can be taken as quantum corrections to the bare metric. Using the quantum stress-energy tensor of the matter fields as a perturbation in the right hand side of the semiclassical Einstein equations, we can obtain a perturbative solution to the backreaction problem up to first order, and determine what changes appear in some important quantities as the mass, the location of the event horizon and the Hawking temperature of the quantum corrected solution.

In this paper we study the effects that vacuum polarization of very massive scalar, vector and spinor fields cause on quasinormal modes of quantum corrected Reissner-Nordström black holes in four dimensions. This is the sequel of our previous work [10] in which we focus on the quantum scalar field case. In the first section we review the Schwinger-DeWitt technique to obtain the one-loop approximation for the effective action for massive fields in the large mass limit, and present the particular results obtained for a classical Reissner-Nordström black hole background. In section II we solve the backreaction problem to obtain the metric that describes the spacetime geometry of an electrically charged semiclassical black hole. Section III is devoted to the calculation of the massless test scalar quasinormal frequencies in this semiclassical background, by sixth order WKB method. Finally in Section IV we give the concluding remarks and comment on related problems to be studied.

In the following we use for the Riemann tensor, its contractions, and the covariant derivatives the sign conventions of Misner, Thorne and Wheeler [24]. Our units are such that $\hbar = c = G = 1$.

II. RENORMALIZED STRESS ENERGY TENSOR FOR QUANTUM MASSIVE FIELDS

In the following we consider the quantization of massive scalar, vector and spinor fields in the large mass limit. The results for the massive scalar field can be found in our previous works [10, 15], and for this reason we will be concerned only with the vector and spinor cases. The action for a single massive vector field A_μ with mass m_v in some generic curved spacetime in four dimensions is

$$S_v = - \int d^4x \sqrt{-g} \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_v^2 A_\mu A^\mu \right) . \quad (1)$$

The equation of motion for the field have the form

$$\hat{V}_\nu^\mu (\nabla) A_\mu = 0 \quad , \quad (2)$$

where the second order operator $\hat{V}_\nu^\mu (\nabla)$ is given by

$$\hat{V}_\nu^\mu (\nabla) = \delta_\nu^\mu \square - \nabla_\nu \nabla^\mu - R_\nu^\mu - m_v^2 \delta_\nu^\mu \quad , \quad (3)$$

where $\square = g^{\mu\nu} \nabla_\mu \nabla_\nu$ is the covariant D'Alembert operator, ∇_μ is the covariant derivative.

For single massive neutral spinor field the action is:

$$S_f = \frac{i}{2} \int d^4x \sqrt{-g} \tilde{\phi} [\gamma^\mu \nabla_\mu \phi + m_f \phi] \quad . \quad (4)$$

In the above expression, ϕ provides a spin representation of the vierbein group and $\tilde{\phi} = \phi^* \gamma$, where $*$ means transpose. The Dirac matrices γ and γ^μ satisfy the usual relation $[\gamma^\mu, \gamma^\nu]_+ = 2g_{\mu\nu} \hat{I}$, where \hat{I} is the 4×4 unit matrix.

The covariant derivative of any spinor ζ obey the commutation relations [23, 25]

$$\nabla_\mu \nabla_\nu \zeta - \nabla_\nu \nabla_\mu \zeta = \frac{1}{2} \mathfrak{F}_{[\alpha, \beta]} R^{\alpha\beta}_{\mu\nu} \zeta \quad , \quad (5)$$

$$\nabla_\nu \nabla_\sigma \nabla_\mu \zeta - \nabla_\sigma \nabla_\nu \nabla_\mu \zeta = \frac{1}{2} \mathfrak{F}_{[\alpha, \beta]} R^{\alpha\beta}_{\mu\sigma} \nabla_\nu \zeta + \nabla_\rho \zeta R_\mu{}^\rho{}_{\nu\sigma} \quad , \quad (6)$$

$$\begin{aligned} \nabla_\sigma \nabla_\tau \nabla_\nu \nabla_\mu \zeta - \nabla_\tau \nabla_\sigma \nabla_\nu \nabla_\mu \zeta &= \frac{1}{2} \mathfrak{F}_{[\alpha, \beta]} R^{\alpha\beta}_{\sigma\tau} \nabla_\nu \nabla_\mu \zeta + \nabla_\nu \nabla_\rho \zeta R_\mu{}^\rho{}_{\sigma\tau} \\ &\quad + \nabla_\rho \nabla_\mu \zeta R_\nu{}^\rho{}_{\sigma\tau} \quad , \end{aligned} \quad (7)$$

and so forth, where $\mathfrak{F}_{[\alpha, \beta]} = \frac{1}{4} [\gamma_\alpha, \gamma_\beta]_-$ are the generators of the vierbein group, $[\ , \]_-$ is the commutator bracket, and $R^{\alpha\beta}_{\mu\nu} = h^\alpha_\sigma h^\beta_\tau R^{\sigma\tau}_{\mu\nu}$, with h^α_β the vierbein which satisfies $h_{\alpha\mu} h^\alpha_\nu = g_{\mu\nu}$. The covariant derivatives of γ , γ^μ and $\mathfrak{F}_{[\alpha, \beta]}$ vanishes. The equation of motion for the field ϕ derived from the action (4) reads

$$(\gamma^\mu \nabla_\mu + m_f) \phi = 0 \quad . \quad (8)$$

The operator \hat{D}_f that gives the evolution of the spinor function in (8) is:

$$\hat{D}_f = \gamma^\mu \nabla_\mu + m_f \quad . \quad (9)$$

The usual formalism of Quantum Field Theory gives an expression for the effective action of the quantum fields A_β , ϕ as a perturbative expansion,

$$\Gamma(A_\beta, \phi) = S(A_\beta, \phi) + \sum_{k \geq 1} \Gamma_{(k)}(A_\beta, \phi) \quad , \quad (10)$$

where $S(A_\beta, \phi)$ is the classical action of the free fields. The one loop contribution of the fields A_β , ϕ to the effective action is expressed in terms of the operators (3) and (9) as:

$$\Gamma_{(1)} = \frac{i}{2} \ln \left(\mathfrak{Det} \hat{V} \right) + \frac{i}{2} \ln \left(\mathfrak{Det} \hat{D} \right) \quad , \quad (11)$$

where $\mathfrak{Det} \hat{F} = \exp(\text{Tr} \ln \hat{F})$ is the functional Berezin superdeterminant of the operator \hat{F} , and $\text{Tr} \hat{F} = (-1)^i F^i_i = \int d^4x (-1)^A F^A_A(x)$ is the functional supertrace [21]. If the

Compton's wavelength of the field is less than the characteristic radius of spacetime curvature [15, 17, 18, 21–23, 26], we can develop an expansion of the above effective action in powers of the inverse square mass of the field. This is known as the Schwinger-DeWitt approximation, and can be applied to "minimal" second order differential operator of the general form

$$\hat{K}_\nu^\mu(\nabla) = \delta_\nu^\mu \square - m^2 \delta_\nu^\mu + Q_\nu^\mu \quad , \quad (12)$$

where $Q_\nu^\mu(x)$ is some arbitrary matrix playing the role of the potential.

Unfortunately, this is not the case of operators (3) and (9). In the case of (3), the presence of the nondiagonal term turn it to be a nonminimal operator.

By fortune we can put (3) as function of some minimal operators, if we note that it satisfies the identity $\hat{V}_\nu^\mu(\nabla)(m_v^2 \delta_\nu^\mu - \nabla_\nu \nabla^\mu) = m_v^2(\delta_\nu^\mu \square - R_\nu^\mu - m_v^2 \delta_\nu^\mu)$. Then the one loop effective action for the nonminimal operator (3) omitting an inessential constant can be written as

$$\frac{i}{2} \text{Tr} \ln \hat{V}_\nu^\mu(\nabla) = \frac{i}{2} \text{Tr} (\delta_\nu^\mu \square - R_\nu^\mu - m_v^2 \delta_\nu^\mu) - \frac{i}{2} \text{Tr} (m_v^2 \delta_\nu^\mu - \nabla_\nu \nabla^\mu) \quad . \quad (13)$$

We can see in (13) that the first term is the effective action of a minimal second order operator $K_\nu^\mu(\nabla)$ with potential $-R_\nu^\mu$. The second term can be transformed as $\text{Tr} \left[\frac{1}{m_v^2} \nabla^\mu \nabla_\nu \right]^n = \text{Tr} \left[\frac{1}{m_v^2} \nabla^\mu \square^{n-1} \nabla_\nu \right] = \text{Tr} \left[\frac{1}{m_v^2} \square \right]^n$ and

$$\frac{i}{2} \text{Tr} (m_v^2 \delta_\nu^\mu - \nabla_\nu \nabla^\mu) = \frac{i}{2} \text{Tr} (m_v^2 - \square) \quad . \quad (14)$$

Then, the effective action for the massive vector field is equal to the effective action of the minimal second order operator $K_\nu^\mu(\nabla)$ minus the effective action of a minimal operator $S_\nu^\mu(\nabla)$ corresponding to a massive scalar field minimally coupled to gravity.

The problem with the Dirac nonminimal operator \hat{D}_f is solved introducing a new spinor variable ψ connected with ϕ by the relation $\phi = \gamma^\sigma \nabla_\sigma \psi - m_f \psi$ so that (8) take the form $\gamma^\mu \gamma^\nu \nabla_\mu \nabla_\nu \psi - m_f^2 \psi = 0$. Making use of the identity $\gamma^\mu \gamma^\nu \nabla_\mu \nabla_\nu = \hat{I} (\square - \frac{1}{4} R)$ equation (8) becomes of the form

$$\hat{D}_f^{min} \psi \equiv \left(\square - \frac{1}{4} R - m_f^2 \right) \psi = 0 \quad , \quad (15)$$

where the potential matrix can be easily identified as $Q = -\frac{1}{4} R \hat{I}$.

Now using the Schwinger-DeWitt representation for the Green functions of the minimal operators, we can obtain for the renormalized one loop effective action of the quantum massive vector and spinor fields the expression $\Gamma_{(1)ren} = \int d^4x \sqrt{-g} \mathfrak{L}_{ren}$ where the renormalized effective Lagrangian reads:

$$\mathfrak{L}_{ren} = \frac{1}{2(4\pi)^2} \sum_{k=3}^{\infty} \frac{1}{k(k-1)(k-2)} \left[\frac{\text{Tr } a_k^{(1)}(x, x) - \text{Tr } a_k^{(0)}(x, x)}{m_v^{2(k-2)}} + \frac{\text{Tr } a_k^{(\frac{1}{2})}(x, x)}{m_f^{2(k-2)}} \right] , \quad (16)$$

$[a_k^{(1)}] = a_k^{(1)}(x, x')$, $[a_k^{(0)}] = a_k^{(0)}(x, x')$ and $[a_k^{(\frac{1}{2})}] = a_k^{(\frac{1}{2})}(x, x')$, whose coincidence limit appears under the supertrace operation in (16) are the HMDS coefficients for the minimal operators \hat{K} , \hat{S} and \hat{D}_f^{min} respectively. As usual, the first three coefficients of the DeWitt-Schwinger expansion, a_0 , a_1 , and a_2 , contribute to the divergent part of the action and can be absorbed in the classical gravitational action by renormalization of the bare gravitational and cosmological constants.

Restricting ourselves here to the terms proportional to m_v^{-2} , using integration by parts and the elementary properties of the Riemann tensor [15, 17–19, 21, 26], we obtain for the renormalized effective lagrangian in the case of the massive vector field considered in this work

$$\begin{aligned} \mathfrak{L}_{ren} = & \frac{1}{192\pi^2 m_v^2} \left[\frac{9}{28} R_{\mu\nu} \square R^{\mu\nu} - \frac{27}{280} R \square R - \frac{5}{72} R^3 + \frac{31}{60} R R_{\mu\nu} R^{\mu\nu} - \frac{52}{63} R_{\nu}^{\mu} R_{\gamma}^{\nu} R_{\mu}^{\gamma} \right. \\ & + \frac{61}{140} R_{\mu\nu} R^{\mu}_{\sigma\gamma\varrho} R^{\nu\sigma\gamma\varrho} - \frac{19}{105} R^{\mu\nu} R_{\gamma\varrho} R_{\mu}^{\gamma} R_{\nu}^{\varrho} - \frac{67}{2520} R_{\gamma\varrho}^{\mu\nu} R_{\mu\nu}^{\sigma\tau} R_{\sigma\tau}^{\gamma\varrho} \\ & \left. - \frac{1}{10} R R_{\mu\nu\gamma\varrho} R^{\mu\nu\gamma\varrho} + \frac{1}{18} R_{\mu}^{\gamma} R_{\nu}^{\varrho} R_{\sigma}^{\mu} R_{\tau}^{\nu} R_{\gamma}^{\sigma} R_{\varrho}^{\tau} \right] . \end{aligned} \quad (17)$$

The interested reader can find the general result for the spinor field case, for example, in reference [17]. As we can see, this final expression of the one loop effective for the massive vector field only differ from that of the massive scalar and spinor fields in the numerical coefficients in front of the purely geometric terms. For $\langle T_{\mu\nu} \rangle_{ren}$ we obtain a very cumbersome expression that, as in the case of (17), is different from that obtained for scalar and spinor fields only in the numerical coefficients that appears in front of the purely geometrical terms. For this reason we not put this very long expression for the stress tensor here and refers the readers to our previous papers [15, 17, 18, 26].

For the present work we deal with the Reissner-Nordström spacetime. The obtained results for the components of the stress-tensor are very simple and can be found in reference [18].

For a quantum scalar field $\phi(x)$ with mass m interacting with gravity with non minimal

coupling constant ξ we find

$$\langle T_s \rangle_\nu^\mu = C_\nu^\mu + \left(\xi - \frac{1}{6} \right) D_\nu^\mu \quad , \quad (18)$$

where

$$\begin{aligned} C_t^t &= -\Upsilon_s(1248Q^6 - 810r^4Q^2 + 855M^2r^4 + 202r^2Q^4 - 1878M^3r^3 + 1152Mr^3Q^2 \\ &\quad + 2307M^2r^2Q^2 - 3084MQ^4r), \\ D_t^t &= \Xi(-792M^3r^3 + 360M^2r^4 + 2604M^2Q^2r^2 - 1008MQ^2r^3 - 2712MQ^4 \\ &\quad + 819Q^6 + 728Q^4r^2), \\ C_r^r &= \Upsilon_s(444Q^6 - 1488MQ^2r^3 + 162Q^2r^4 + 842Q^4r^2 - 1932MQ^4r + 315M^2r^4 \\ &\quad + 2127M^2Q^2r^2 - 462M^3r^3), \\ D_r^r &= \Xi(-792M^3r^3 + 360M^2r^4 + 2604M^2Q^2r^2 - 1008MQ^2r^3 - 2712MQ^4r \\ &\quad + 819Q^6 + 728Q^4r^2), \\ C_\theta^\theta &= -\Upsilon_s(3044Q^4r^2 - 2202M^3r^3 - 10356MQ^4r + 3066Q^6 - 4884MQ^2r^3 \\ &\quad + 9909M^2Q^2 + 945M^2r^4 + 486Q^2r^4), \\ D_\theta^\theta &= \Xi(3276M^2Q^2r^2 - 1176MQ^2r^3 - 3408MQ^4r + 1053Q^6 - 1008M^3r^3 \\ &\quad + 432M^2r^4 + 832Q^4r^2), \end{aligned}$$

For the massive vector field we obtain:

$$\begin{aligned} \langle T_v \rangle_t^t &= -\Upsilon_v(-31057Q^6 - 12150r^4Q^2 - 1665M^2r^4 - 41854r^2Q^4 - 93537M^2r^2Q^2 \\ &\quad + 3666M^3r^3 + 69024Mr^3Q^2 + 10751MQ^4r) \quad , \\ \langle T_v \rangle_r^r &= \Upsilon_v(-10448MQ^2r^3 + 2430Q^2r^4 + 6442Q^4r^2 - 693M^2r^4 + 12907M^2Q^2r^2 \\ &\quad + 5365Q^6 - 16996MQ^4r + 1050M^3r^3) \quad , \\ \langle T_v \rangle_\theta^\theta &= -\Upsilon_v(20908Q^4r^2 + 4854M^3r^3 - 44068MQ^4r - 31708MQ^2r^3 + 7290Q^2r^4 \\ &\quad + 30881M^2Q^2r^2 - 2079M^2r^4 + 13979Q^6) \quad , \end{aligned}$$

and for the spinor field the resulting components are given by

$$\begin{aligned} \langle T_f \rangle_t^t &= -\Upsilon_f(-21496rMQ^4 + 4917Q^6 + 10544r^2Q^4 - 22464Mr^3Q^2 + 21832M^2r^2Q^2 \\ &\quad - 1080M^2r^4 + 2384M^3r^3 + 5400r^4Q^2) \quad , \\ \langle T_f \rangle_r^r &= \Upsilon_f(-6336Mr^3Q^2 + 8440M^2r^2Q^2 + 2253Q^6 + 3560r^2Q^4 - 8680rMQ^4 \\ &\quad + 504M^2r^4 - 784M^3r^3 + 1080r^4Q^2) \quad , \\ \langle T_f \rangle_\theta^\theta &= -\Upsilon_f(12080r^2Q^4 - 33984rMe^4 + 9933Q^6 + 30808M^2r^2Q^2 - 20016Mr^3Q^2 \\ &\quad + 1512M^2r^4 - 3536M^3r^3 + 3240r^4Q^2) \quad . \end{aligned}$$

In the above expressions, we have $\Upsilon_s = (30240\pi^2 m^2 r^{12})^{-1}$, $\Upsilon_v = (10080\pi^2 m_v^2 r^{12})^{-1}$, $\Upsilon_f =$

$(40320\pi^2 m_j^2 r^{12})^{-1}$ and $\Xi = (720\pi^2 m^2 r^{12})^{-1}$. Q and M denotes the charge and bare mass of the black hole.

The above quantum stress energy tensors are regular at the event horizon, as is to be expected due to the local nature of the Schwinger-DeWitt approximation and the regular nature of the horizon. Also from the general form of the geometric terms conforming the general expression for the constructed stress tensor, we see that it is covariantly conserved, thus indicating that it is a good candidate for the expected exact one in our large mass approximation.

In the specific case of a Reissner-Nordström spacetimes, Anderson *et.al* showed, using detailed numerical results for the scalar field case, that for $m_s M \geq 2$ the deviation of the approximate stress energy tensor from the exact one lies within a few percent [14]. The more general condition for the validity of the Schwinger-DeWitt approximation in the case of a spin- j field can be written as $m_j M \geq 1$, where m_j and M are respectively the field and black hole masses. In the following we carefully take into account the above condition in numerical calculations.

III. SEMICLASSICAL SOLUTION

In our previous paper we have shown how to find the general solution to the backreaction problem in spacetimes with spherical symmetry [10]. In the following we solve the general semiclassical Einstein equations assuming that there are an electromagnetic field as a classical source, and multiple quantized massive free fields as a perturbative quantum source, so the solution to the backreaction problem gives a quantum corrected Reissner-Nordström black hole.

In this limit in which we deal with free fields on a background spacetime, the treatment of the backreaction due to a collection of fields with different spins is easy. This is due to the fact that the quantum stress tensors in this limit only depends quadratically on the fields and are flavour diagonal. For example, for a set of N_s real scalars, upon renormalization we have

$$\langle T_\mu^\nu \rangle_{ren}^s = \sum_{k=1}^{N_s} \langle (T_\mu^\nu)_k \rangle_{ren}^s = N_s \langle (T_\mu^\nu) \rangle_{ren}^s \quad , \quad (19)$$

where (T_μ^ν) is the classical stress tensor for a single scalar. The last equality above follows

from the fact that the renormalization procedure is independent of the species label k . In a similar manner we can arrive to the same conclusion for the renormalization of spinor and vector field stress tensors.

The above statements permit us to obtain a good approximation to the multiple field backreaction using as the source term in the semiclassical Einstein equations an appropriate weighted combination (with weights N_s , N_v and N_f) of the single-species renormalized stress energy tensors.

In the case of our interest the general form for the line element that solves the backreaction problem, considering only terms that are linear in the perturbation parameter $\varepsilon = 1/M^2$, is given by

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad , \quad (20)$$

with

$$\frac{1}{B(r)} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{8\pi}{r} \sum_j N_j \int_{-\infty}^r \zeta^2 \langle T_t^t \rangle_j d\zeta \quad , \quad (21)$$

and

$$A(r) = \frac{1}{B(r)} \prod_j \exp \{ \lambda_j(r) \} \quad , \quad (22)$$

where

$$\lambda_j(r) = 8\pi N_j \int_{-\infty}^r \zeta B(\zeta) \left(\langle T_r^r \rangle_j - \langle T_t^t \rangle_j \right) d\zeta \quad . \quad (23)$$

where the subindex j denotes the different single spin species considered (scalar, vector and spinor field). Inserting the corresponding expressions for the temporal component of the quantum stress tensor for the different fields considered in this work, we obtain

$$\frac{1}{B(r)} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{\varepsilon}{\pi} \sum_j \frac{N_j}{m_j^2} F_j(r) \quad , \quad (24)$$

where, for the scalar case

$$F_s(r) = E(r) + \xi H(r) \quad , \quad (25)$$

with

$$\begin{aligned} E(r) &= -\frac{613M^3Q^4}{840r^9} + \frac{2327M^2Q^6}{1134r^{10}} - \frac{3M^2Q^2}{70r^6} + \frac{5M^4}{28r^6} - \frac{1237M^5}{3780r^7} + \frac{883M^2Q^4}{4410r^8} \\ &\quad - \frac{82M^3Q^2}{315r^7} + \frac{1369M^4Q^2}{1764r^8}, \\ H(r) &= \frac{28M^3Q^2}{15r^7} + \frac{113M^3Q^4}{30r^9} - \frac{91M^2Q^6}{90r^{10}} - \frac{52M^2Q^4}{45r^8} - \frac{4M^4}{5r^6} + \frac{22M^5}{15r^7} - \frac{62M^4Q^2}{15r^8}. \end{aligned}$$

$$\lambda_s(r) = \frac{\varepsilon}{\pi m^2} \left(\frac{184M^3Q^2}{441r^7} - \frac{29M^4}{140r^6} - \frac{229M^2Q^4}{840r^8} + \frac{M^2Q^2}{35r^6} \right) + \frac{\varepsilon\zeta}{\pi m^2} \left(\frac{14M^4}{15r^6} + \frac{13M^2Q^4}{10r^8} - \frac{32M^3Q^2}{15r^7} \right).$$

and for vector and spinor fields

$$\begin{aligned} F_v(r) &= \frac{26879}{2520} \frac{M^3Q^4}{r^9} + \frac{2876}{315} \frac{M^3Q^2}{r^7} + \frac{611}{1260} \frac{M^5}{r^7} - \frac{37}{140} \frac{M^4}{r^6} - \frac{20927}{4410} \frac{M^2Q^4}{r^8} \\ &\quad - \frac{10393}{980} \frac{M^4Q^2}{r^8} - \frac{27}{14} \frac{M^2Q^2}{r^6} - \frac{31057}{11340} \frac{M^2Q^6}{r^{10}}, \\ F_f(r) &= \frac{2687}{5040} \frac{M^3Q^4}{r^9} - \frac{1639}{15120} \frac{M^2Q^6}{r^{10}} - \frac{149}{1890} \frac{M^5}{r^7} \\ &\quad - \frac{3}{14} \frac{M^2Q^2}{r^6} + \frac{3}{70} \frac{M^4}{r^6} + \frac{26}{35} \frac{M^3Q^2}{r^7} - \frac{2729}{4410} \frac{M^4Q^2}{r^8} - \frac{659}{2205} \frac{M^2Q^4}{r^8}, \end{aligned} \quad (26)$$

$$\begin{aligned} \lambda_v(r) &= \frac{\epsilon}{\pi m_v^2} \left(\frac{131}{420} \frac{M^4}{r^6} + \frac{9}{7} \frac{M^2Q^2}{r^6} - \frac{9784}{2205} \frac{M^3Q^2}{r^7} + \frac{2141}{840} \frac{M^2Q^4}{r^8} \right) \\ \lambda_f(r) &= \frac{\epsilon}{\pi m_f^2} \left(\frac{37}{560} \frac{M^2Q^4}{r^8} - \frac{11}{210} \frac{M^4}{r^6} + \frac{1}{7} \frac{M^2Q^2}{r^6} - \frac{52}{245} \frac{M^3Q^2}{r^7} \right). \end{aligned}$$

The horizon for the quantum corrected solution will be, up to first order in the perturbation parameter, at position r_+ given by

$$r_+ = r_H \left(1 + \sum_j N_j \Lambda_j \right), \quad (27)$$

where

$$\Lambda_j = -\frac{4\pi}{(M - Q^2/r_H)} \int_{\infty}^{r_h} \zeta^2 \langle T_t^t(\zeta) \rangle_j d\zeta. \quad (28)$$

and r_H is the position of the event horizon of the bare classical solution. Upon substitution of the required quantities in the above expression we find

$$\Lambda_j = \frac{\varepsilon \Gamma_j}{\pi m_j^2 (M - Q^2/r_H)}, \quad (29)$$

with

$$\Gamma_s = \Theta + \xi \Omega, \quad (30)$$

with

$$\begin{aligned} \Theta &= \frac{613M^3Q^4}{1680r_H^8} - \frac{2327M^2Q^6}{22680r_H^9} + \frac{3M^2Q^2}{140r_H^5} - \frac{5M^4}{56r_H^6} + \frac{1237M^5}{7560r_H^6} - \frac{883M^2Q^4}{8820r_H^7} \\ &\quad + \frac{41M^3Q^2}{315r_H^6} - \frac{1369M^4Q^2}{3528r_H^7}, \\ \Omega &= -\frac{14M^3Q^2}{15r_H^6} - \frac{113M^3Q^4}{60r_H^8} + \frac{91M^2Q^6}{180r_H^9} + \frac{26M^2Q^4}{45r_H^7} + \frac{2M^4}{5r_H^5} - \frac{11M^5}{15r_H^6} + \frac{31M^4Q^2}{15r_H^7}. \end{aligned}$$

and for the vector and fermion components we find

$$\Gamma_v = \frac{27}{28} \frac{M^2 Q^2}{r_H^5} + \frac{37}{280} \frac{M^4}{r_H^5} - \frac{26879}{5040} \frac{M^3 Q^4}{r_H^8} - \frac{1438}{315} \frac{M^3 Q^2}{r_H^6} + \frac{31057}{22680} \frac{M^2 Q^6}{r_H^9} - \frac{611}{2520} \frac{M^5}{r_H^6} + \frac{20927}{8820} \frac{M^2 Q^4}{r_H^7} + \frac{10393}{1960} \frac{M^4 Q^2}{r_H^7} , \quad (31)$$

$$\Gamma_f = -\frac{2687}{10080} \frac{M^3 Q^4}{r_H^7} + \frac{149}{3780} \frac{M^5}{r_H^5} - \frac{3}{140} \frac{M^4}{r_H^4} + \frac{3}{28} \frac{M^2 Q^2}{r_H^4} + \frac{2729}{8820} \frac{M^4 Q^2}{r_H^6} - \frac{13}{35} \frac{M^3 Q^2}{r_H^5} + \frac{659}{4410} \frac{M^2 Q^4}{r_H^6} + \frac{1639}{30240} \frac{M^2 Q^6}{r_H^8} . \quad (32)$$

IV. SCALAR PERTURBATIONS AND QUASINORMAL MODES

In the following we consider the evolution of a test massless scalar field $\Phi(x^\mu)$ with $x^\mu = (t, r, \theta, \phi)$, in the quantum corrected gravitational background studied above. The dynamics of for this test field is governed by the Klein-Gordon equation

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial \Phi}{\partial x^\nu} \right) = 0 , \quad (33)$$

with $g_{\mu\nu}$ is the metric tensor of semiclassical solution and g its determinant. Upon separation of the angular and radial part in the above equation and the introduction of the radial tortoise coordinate

$$\frac{d^2}{dr_*^2} Z_L - [\omega^2 - V] Z_L = 0 , \quad (34)$$

where $Z_L(r)$ denotes the radial component of the wave function, ω is the quasinormal frequency and V is the effective potential. The potential V is a function of the metric components $g_{\mu\nu}$ and the multipolar number L , and for the test massless scalar field considered in this work, is given by the general expression

$$V[r(r_*)] = A(r) \frac{L(L+1)}{r^2} + \frac{A(r)}{2rB(r)} [(\ln A(r))' - (\ln B(r))'] , \quad (35)$$

where the prime refers to the derivative with respect to the radial coordinate r . For semiclassical black holes we have in general the following expression for the effective potential

$$V(r) = V^c(r) + \frac{\varepsilon}{\pi} U(r) + O(\varepsilon^2) , \quad (36)$$

where $V^c(r)$ is the scalar effective potential of the bare black hole solution and $U(r)$ is a complicated function of the contributions $\frac{N_j}{m_j^2} U_j(r)$ due to the vacuum polarization effect

related the multiple field backreaction, that we will not write here. In the case of a classical Reissner-Nordström black hole $V^c(r)$ is given by

$$V^c(r) = \frac{(r^2 - 2Mr + Q^2)(-2Q^2 + \beta r^2 + 2Mr)}{r^6}, \quad (37)$$

where $\beta = L(L+1)$. For the semiclassical black hole solution considered in this paper, where the vacuum polarization effects comes from the quantization of massive scalar, vector and spinor fields in the large mass limit, the particular expression for $U_j(r)$ results

$$U_s(r) = W_1(r) + \xi W_2(r), \quad (38)$$

where

$$\begin{aligned} W_1(r) = & -\frac{1751}{4410} \frac{M^2 Q^4}{r^{10}} - \frac{9}{20} \frac{M^4}{r^8} + \frac{1021}{540} \frac{M^5}{r^9} - \frac{1816}{945} \frac{M^6}{r^{10}} + \frac{6}{35} \frac{M^2 Q^2}{r^8} \\ & + \frac{674641}{158760} \frac{M^3 Q^6}{r^{13}} + \frac{17}{105} \frac{M^3 Q^2}{r^9} - \frac{13271}{1764} \frac{M^4 Q^4}{r^{12}} - \frac{625}{756} \frac{M^2 Q^8}{r^{14}} \\ & - \frac{23353}{15876} \frac{M^2 Q^6}{r^{12}} + \frac{8559}{1960} \frac{M^3 Q^4}{r^{11}} - \frac{962}{245} \frac{M^4 Q^2}{r^{10}} + \frac{16687}{2940} \frac{M^5 Q^2}{r^{11}} \\ & + L(L+1) \left(-\frac{1529}{22680} \frac{M^2 Q^6}{r^{12}} - \frac{1}{35} \frac{M^4}{r^8} - \frac{1}{70} \frac{M^2 Q^2}{r^8} \right. \\ & \left. + \frac{47}{540} \frac{M^5}{r^9} - \frac{773}{17640} \frac{M^2 Q^4}{r^{10}} + \frac{44}{441} \frac{M^3 Q^2}{r^9} + \frac{821}{3528} \frac{M^3 Q^4}{r^{11}} - \frac{1171}{4410} \frac{M^4 Q^2}{r^{10}} \right), \end{aligned}$$

and

$$\begin{aligned} W_2(r) = & L(L+1) \left(-\frac{4}{15} \frac{M^3 Q^2}{r^9} + \frac{16}{15} \frac{M^4 Q^2}{r^{10}} - \frac{29}{30} \frac{M^3 Q^4}{r^{11}} - \frac{2}{5} \frac{M^5}{r^9} + \frac{13}{90} \frac{M^2 Q^4}{r^{10}} \right. \\ & \left. + \frac{13}{45} \frac{M^2 Q^6}{r^{12}} + \frac{2}{15} \frac{M^4}{r^8} \right) + \frac{26}{3} \frac{M^2 Q^6}{r^{12}} - \frac{162}{5} \frac{M^5 Q^2}{r^{11}} - \frac{2101}{90} \frac{M^3 Q^6}{r^{13}} \\ & + \frac{128}{15} \frac{M^6}{r^{10}} + 2 \frac{M^4}{r^8} + \frac{13}{3} \frac{M^2 Q^8}{r^{14}} + \frac{182}{45} \frac{M^2 Q^4}{r^{10}} - \frac{289}{10} \frac{M^3 Q^4}{r^{11}} - \frac{28}{5} \frac{M^3 Q^2}{r^9} \\ & - \frac{42}{5} \frac{M^5}{r^9} + \frac{416}{15} \frac{M^4 Q^2}{r^{10}} + \frac{130}{3} \frac{M^4 Q^4}{r^{12}}. \end{aligned}$$

for the scalar case and

$$\begin{aligned} U_v(r) = & -\frac{107577}{980} \frac{M^5 Q^2}{r^{11}} + \frac{306442}{2205} \frac{M^4 Q^2}{r^{10}} + \frac{34907}{196} \frac{M^4 Q^4}{r^{12}} + \frac{13}{20} \frac{M^4}{r^8} + \frac{54}{7} \frac{M^2 Q^2}{r^8} \\ & - \frac{491}{180} \frac{M^5}{r^9} + \frac{872}{315} \frac{M^6}{r^{10}} - \frac{1205}{21} \frac{M^3 Q^2}{r^9} - \frac{303071}{1960} \frac{M^3 Q^4}{r^{11}} - \frac{15415961}{158760} \frac{M^3 Q^6}{r^{13}} \\ & + \frac{681461}{15876} \frac{M^2 Q^6}{r^{12}} + \frac{29027}{882} \frac{M^2 Q^4}{r^{10}} + \frac{66421}{3780} \frac{M^2 Q^8}{r^{14}} + L(L+1) \left(-\frac{5}{36} \frac{M^5}{r^9} \right. \\ & + \frac{4678}{2205} \frac{M^3 Q^2}{r^9} + \frac{6653}{5880} \frac{M^3 Q^4}{r^{11}} - \frac{16067}{17640} \frac{M^2 Q^4}{r^{10}} - \frac{4307}{22680} \frac{M^2 Q^6}{r^{12}} + \frac{1}{21} \frac{M^4}{r^8} \\ & \left. - \frac{6257}{4410} \frac{M^4 Q^2}{r^{10}} - \frac{9}{14} \frac{M^2 Q^2}{r^8} \right), \end{aligned} \quad (39)$$

and

$$\begin{aligned}
U_f(r) = & -\frac{2279}{2520} \frac{M^2 Q^8}{r^{14}} - \frac{11101}{1470} \frac{M^5 Q^2}{r^{11}} + \frac{24764}{2205} \frac{M^4 Q^2}{r^{10}} + \frac{5092}{441} \frac{M^4 Q^4}{r^{12}} - \frac{191}{35} \frac{M^3 Q^2}{r^9} \\
& - \frac{1}{10} \frac{M^4}{r^8} - \frac{119141}{21168} \frac{M^3 Q^6}{r^{13}} + \frac{113}{270} \frac{M^5}{r^9} + \frac{6}{7} \frac{M^2 Q^2}{r^8} - \frac{80}{189} \frac{M^6}{r^{10}} - \frac{8775}{784} \frac{M^3 Q^4}{r^{11}} \\
& + \frac{23797}{8820} \frac{M^2 Q^4}{r^{10}} + \frac{3569}{1323} \frac{M^2 Q^6}{r^{12}} + L(L+1) \left(\frac{7}{270} \frac{M^5}{r^9} - \frac{1}{105} \frac{M^4}{r^8} + \frac{12}{49} \frac{M^3 Q^2}{r^9} \right. \\
& \left. - \frac{1}{14} \frac{M^2 Q^2}{r^8} - \frac{3173}{35280} \frac{M^2 Q^4}{r^{10}} - \frac{544}{2205} \frac{M^4 Q^2}{r^{10}} - \frac{8}{189} \frac{M^2 Q^6}{r^{12}} + \frac{6659}{35280} \frac{M^3 Q^4}{r^{11}} \right).
\end{aligned} \tag{40}$$

for the vector and spinor cases.

In Figure (1) is presented the effective potential V taking into account, as an example, the backreaction of multiple fields with weights N_s , N_v and N_f all equal to 10.

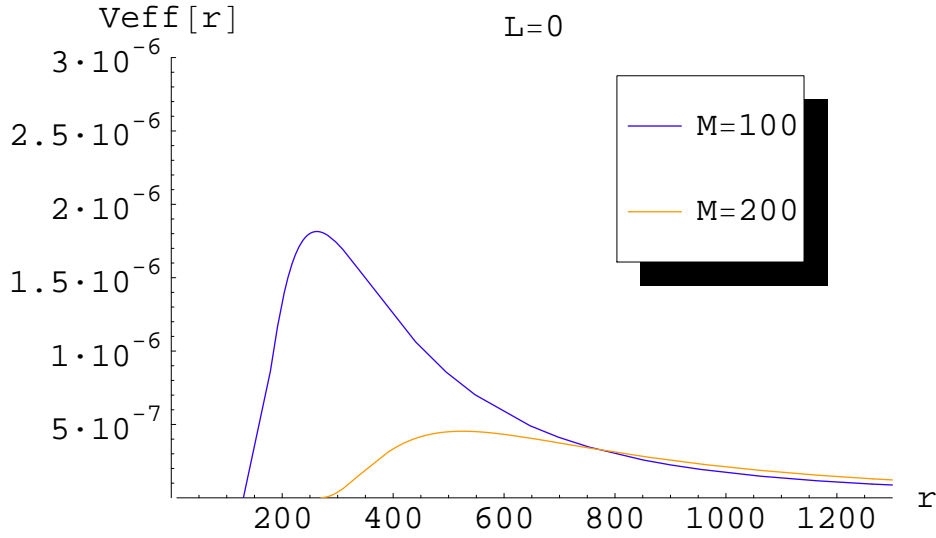


Figure 1. *Effective potential of $L=0$ scalar modes for Semiclassical black hole with $M = 100$ (top curve) and $M = 200$ (bottom curve). $Q/M = 0.95$, $N_s = N_v = N_f = 10$ and the mass parameter of all quantum fields are chosen to be $m=1/10$.*

As it is observed, the figure shows a definite positive potential barrier, i.e, a well behaved function that goes to zero at spatial infinity and gets a maximum value near the event horizon. The quasinormal modes are solutions of the wave equation (34) with the specific boundary conditions requiring pure out-going waves at spatial infinity and pure in-coming waves on the event horizon. The quasinormal frequencies are in general complex numbers,

whose real part determines the real oscillation frequency and the imaginary part determines the damping rate of the quasinormal mode. In order to evaluate the quasinormal modes for the situation considered in this report field, we use the well known WKB technique at sixth order, that can give accurate values of the lowest (that is longer lived) quasinormal frequencies, and was used in several papers for the determination of quasinormal frequencies in a variety of systems [27]. The first order WKB technique was applied to finding quasinormal modes for the first time by Shutz and Will [28]. Latter this approach was extended to the third order beyond the eikonal approximation by Iyer and Will [29] and to the sixth order by Konoplya [30].

The sixth order WKB expansion gives a relative error which is about two orders less than the third WKB order, and allows us to determine the quasinormal frequencies through the formula

$$i \frac{(\omega^2 - V_0)}{\sqrt{-2V_0''}} - \sum_{j=2}^6 \Pi_j = n + \frac{1}{2} \quad , \quad n = 0, 1, 2, \dots \quad (41)$$

where V_0 is the value of the potential at its maximum as a function of the tortoise coordinate, and V_0'' represents the second derivative of the potential with respect to the tortoise coordinate at its peak. The correction terms Π_j depend on the value of the effective potential and its derivatives (up to the $2i$ -th order) in the maximum, see [31] and references therein.

The results of the numerical evaluation of the first two fundamental quasinormal frequencies in the case considered in this work is showed in table (I).

From figures (2) and (3) we see that the backreaction of the quantized multiple fields upon the classical charged black hole gives rise to an increasing of the real oscillation frequencies and to a decreasing of the damping rate, for physically interesting values of the black hole mass. Then, as a consequence of the vacuum polarization effect due to the multiple massive fields, we have an effective increasing of the quality factor, proportional to the ratio $\frac{|Re(\omega)|}{|Im(\omega)|}$. As expected, the differences in the quasinormal frequencies when the black hole mass increases tend to become small. It is interesting to note that the above results show significant differences with those obtained previously considering only the backreaction of a quantized massive scalar field upon the classical charged black hole solution. In that case the quality factor is small for semiclassical black hole with respect to the classical solution, and the shift in the quasinormal frequencies is less pronounced. As the numerical calculations show, this is not the case if the separate backreaction due to massive vector and spinor fields are con-

Semiclassical solution				Classical solution			
$M = 100$							
L	n	$Re(\varpi)$	$-Im(\varpi)$	L	n	$Re(\varpi)$	$-Im(\varpi)$
0	0	10.2421	6.64263	0	0	5.52131	29.4487
1	0	28.7831	6.80833	1	0	27.1215	7.5690
1	1	26.5433	21.0202	1	1	23.2393	33.6626
$M = 120$							
L	n	$Re(\varpi)$	$-Im(\varpi)$	L	n	$Re(\varpi)$	$-Im(\varpi)$
0	0	8.53509	5.53553	0	0	4.6011	24.5404
1	0	23.9859	5.6736	1	0	22.2679	6.30758
1	1	22.1194	17.0889	1	1	19.3661	28.0521
$M = 150$							
L	n	$Re(\varpi)$	$-Im(\varpi)$	L	n	$Re(\varpi)$	$-Im(\varpi)$
0	0	6.82809	4.42843	0	0	3.68089	19.6324
1	0	19.1887	4.5389	1	0	19.0144	5.04608
1	1	17.6956	14.0135	1	1	15.4929	22.4418

Table I. *Rescaled scalar quasinormal frequencies $\varpi = 10^3\omega$ for the classical and semiclassical Reissner-Nordström black hole, with $Q/M = 0.95, N_s = N_v = N_f = 10$ and $m = 1/10$.*

sidered. In each one of this cases, we obtain an increasing of the quality factor more relevant than the decreasing showed by the scalar field case. As a result, the combination of all types of fields give a higher quality factor with respect to the bare black hole. Thus, we arrive to the conclusion that the semiclassical Reissner-Nordström black holes are better oscillators than its classical partners. Then, in this case, there is a qualitative correspondence with the results obtained by Konoplya in reference [11] for the BTZ black hole dressed by a quantum conformal massless scalar field, where he studied the backreaction due to Hawking radiation upon the classical spacetime.

We also studied the dependence of the quasinormal frequencies for a fixed black hole bare mass and different values of the the quantum field mass m_j , obtaining similar results with respect to the massive scalar field case: a little dependance of the quasinormal frequencies

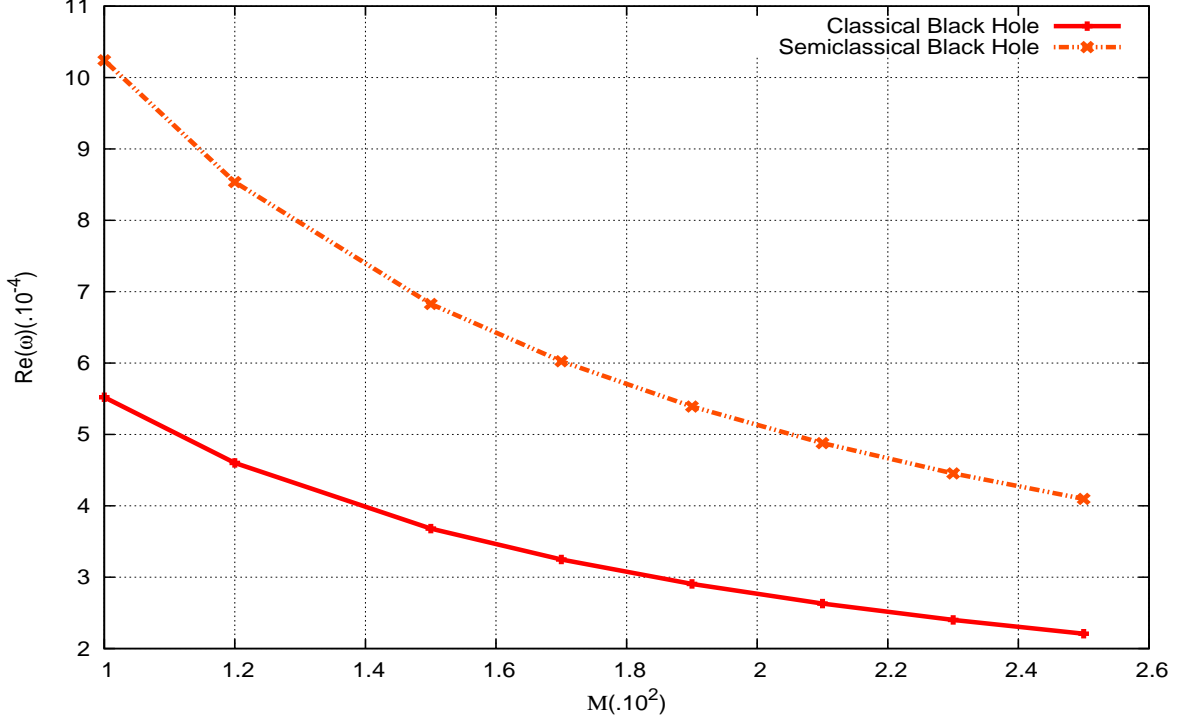


Figure 2. Dependence of $Re(\omega)$ on M for classical and semiclassical black holes. The parameters are chosen to be $Q/M = 0.95$, $m = 1/10$, $L = 0$, $N_s = N_v = N_f = 10$ and $n = 0$.

on this parameters. As the quantum field mass increases, we found a very small increment in the real part of the frequencies for semiclassical black holes, and a very small decrease in the imaginary part. The same occur if we consider the variation of the coupling constant between the massive scalar field component and the gravitational background: the quasinormal frequencies are insensitive to the variation of this parameter. Therefore, the shift in the quasinormal frequencies with respect to the classical bare black hole appears to be the same for the given range of the quantum field masses. With respect to the multiplicity number N_j of a given quantum field we find that, as expected, the shift shows some increment as this parameters increases, an effect that is more pronounced for very large N_j . It is important to take care with the fact that, very large values of this numbers can imply that the total quantum stress tensor becomes a large quantity, such that the perturbative treatment of the backreaction problem used here becomes inadequate. By fortune, this is not the case for physically interesting values of N_j .

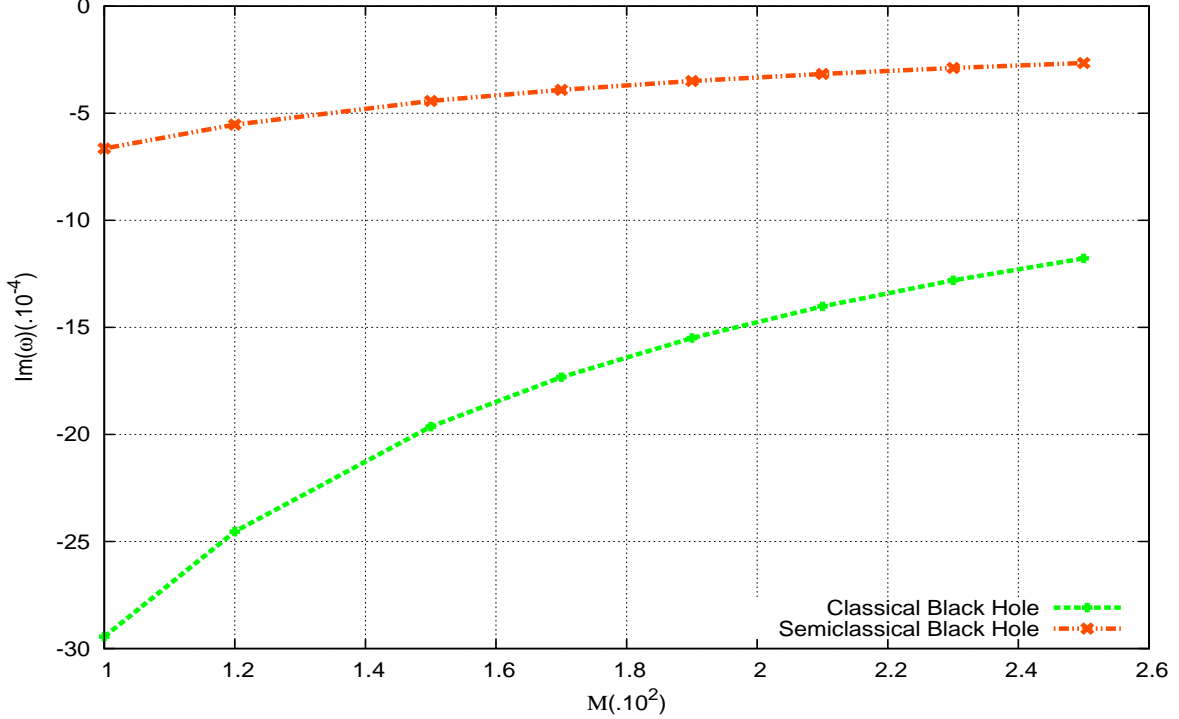


Figure 3. Dependence of $\text{Im}(\omega)$ on M for classical and semiclassical black holes. The parameters are chosen to be $Q/M = 0.95$, $m = 1/10$, $L = 0$, $N_s = N_v = N_f = 10$ and $n = 0$.

V. CONCLUDING REMARKS

We have studied the influence of the backreaction due to vacuum polarization of multiple species of large mass quantum massive fields, belonging to the standard model, upon the structure of scalar quasinormal frequencies for semiclassical charged black holes. The effect observed is a shift in the quasinormal frequencies the semiclassical solution, such that quantum corrected black holes becomes better oscillators that classical ones. It is important to verify that the above effects are also true for the quasinormal ringing phase in the evolution of spinor and electromagnetic test fields. We are currently investigating such problems and the results will be presented in future reports.

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